

# Package ‘BsplineQuantReg’

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**Type** Package

**Title** 'Constrained Quantile Regression with B-Splines'

**Version** 0.2.1

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**Description** Quantile regression with B-splines under shape constraints.  
The initial version with cubic splines is now augmented with splines of degree 1 to 4. Constraints for degrees 3 (monotone) and 4 (monotone and convex) use the Karlin-Studden SOCP characterization for the sign of the polynomial, while other constraints applied at the knots are added as linear problems. The method for cubic splines is described in Abbes (2026) <[doi:10.5281/zenodo.17427913](https://doi.org/10.5281/zenodo.17427913)>. Other formulations are simple consequences of the other given references. This R implementation is intended for demonstration and prototyping. All B-spline and polynomial functions have been rewritten for consistency. An equivalent Python package is available at <<https://pypi.org/project/BsplineQuantRegpy/>>.

**License** GPL-3

**URL** <https://github.com/alexandreabbes/BsplineQuantReg>,  
<https://doi.org/10.5281/zenodo.17427913>

**BugReports** <https://github.com/alexandreabbes/BsplineQuantReg/issues>

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apply\_karlin\_quadratic

*Apply Karlin-Studden constraints for a quadratic polynomial*

---

### Description

For a quadratic polynomial  $p(u) = a*u^2 + b*u + c$  on  $[0,1]$ , this function applies the Karlin-Studden SOCP constraints to ensure  $p(u) \geq 0$  or  $p(u) \leq 0$  on  $[0,1]$ .

### Usage

apply\_karlin\_quadratic(p2, p1, p0, z0, sign = 1)

**Arguments**

|      |                                                             |
|------|-------------------------------------------------------------|
| p2   | Coefficient of $u^2$                                        |
| p1   | Coefficient of $u$                                          |
| p0   | Constant term                                               |
| z0   | Auxiliary SOCP variable                                     |
| sign | Sign of the constraint (+1 for $\geq 0$ , -1 for $\leq 0$ ) |

**Value**

List of CVXR constraints

---

|              |                                                                                                                                                                                                                                       |
|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Bspline_base | <i>Build B-spline basis in piecewise polynomial form Computes local polynomial coefficients for each B-spline basis function on each interval. Polynomials are expressed in the canonical basis Uses De Boor's recursion formula.</i> |
|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

---

**Description**

Build B-spline basis in piecewise polynomial form Computes local polynomial coefficients for each B-spline basis function on each interval. Polynomials are expressed in the canonical basis Uses De Boor's recursion formula.

**Usage**

```
Bspline_base(sn, degree = 3, der = NULL, verbose = FALSE)
```

**Arguments**

|         |                                                                                                                                                                                                                                                              |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| sn      | Extended knot vector (including endpoint repetitions) This means if $t_0..t_{kn}$ is the set of knot then sn should be given as a vector with "degree" times $t_0$ and $t_{kn}$ at the beginning and the ends. its length is number of intervals+1+2*degree. |
| degree  | B-spline degree (default = 3 for cubic)                                                                                                                                                                                                                      |
| der     | Derivative order (0 = original basis)                                                                                                                                                                                                                        |
| verbose | boolean FALSE (default) or TRUE.                                                                                                                                                                                                                             |

**Value**

A list containing:

|      |                                                                                                                                                                                                                                                                                                                                                                                       |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| base | Coefficients in the local bases, in the form of an 3-d array $[j, nu, coeff]$ , $j$ : the number of the spline in the basis, $nu$ : the number of the interval in the extended notation, $coeff$ : the coefficients in decreasing order convention on the local bases $(t-s_{nu})^l$ , $l=3..1$ $base[j,,]$ is a matrix of piecewise polynomial function compatible with the pp-form. |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

|             |                                                                                                                           |
|-------------|---------------------------------------------------------------------------------------------------------------------------|
| base0       | Coefficients in canonical basis (centered at 0) (1, t, t <sup>2</sup> , t <sup>3</sup> ) centered at the interval origin. |
| ext_knot    | Extended knot vector                                                                                                      |
| knot        | Effective knot partition including ends)                                                                                  |
| degree      | Spline degree                                                                                                             |
| n_splines   | Number of basis functions                                                                                                 |
| deriv_order | Applied derivative order                                                                                                  |

### Examples

```
sn <- c(0,0,0,0,1,2,3,4,5,5,5,5)
basis <- Bspline_base(sn, degree=3)
x=(0:(5*100))/100
y=bs_direct(basis,x)
matplot(x,t(y))
#or simple:
#view_basis(basis)
```

---

|               |                                       |
|---------------|---------------------------------------|
| Bspline_deriv | <i>Differentiate a B-spline basis</i> |
|---------------|---------------------------------------|

---

### Description

Computes the basis of order der derivatives of a B-spline basis.

### Usage

```
Bspline_deriv(bspline, der = 2, verbose = FALSE)
```

### Arguments

|         |                                  |
|---------|----------------------------------|
| bspline | Object returned by Bspline_base  |
| der     | Derivative order                 |
| verbose | boolean FALSE (default) or TRUE. |

### Value

A list similar to Bspline\_base for the derivative basis

---

bspline\_to\_deriv\_coeffs\_cubic

*normalized first and second derivative coefficients on each interval.  
for a cubic B-spline basis*

---

### Description

normalized first and second derivative coefficients on each interval. for a cubic B-spline basis

### Usage

```
bspline_to_deriv_coeffs_cubic(tn, degree = 3, x_values = 0, verbose = FALSE)
```

### Arguments

|          |                                                         |
|----------|---------------------------------------------------------|
| tn       | Knot vector (effective partition, not extended)         |
| degree   | Spline degree (default = 3)                             |
| x_values | Evaluation points for design matrix (0 = no evaluation) |
| verbose  | boolean FALSE (default) or TRUE.                        |

### Value

A list containing (kn: Nb intervals, N=kn+3: Nb basis functions):

|    |                                                                                  |
|----|----------------------------------------------------------------------------------|
| d0 | Design matrix (if x_values provided)                                             |
| d1 | First derivative coefficients [a3, a2, a1] for each interval: (kn x N x 3) array |
| d2 | Second derivative values at knot: ((kn+1) x N) matrix                            |
| d3 | Third derivative values at knots: (kn x N) matrix                                |

---

bspline\_to\_deriv\_coeffs\_lin

*Derivative coefficients for linear B-spline to Computes normalized  
first derivative coefficients for linear B-splines. For linear splines, the  
derivative is constant on each interval.*

---

### Description

Derivative coefficients for linear B-spline to Computes normalized first derivative coefficients for linear B-splines. For linear splines, the derivative is constant on each interval.

### Usage

```
bspline_to_deriv_coeffs_lin(tn, degree = 1, x_values = 0, verbose = FALSE)
```

**Arguments**

|          |                                                         |
|----------|---------------------------------------------------------|
| tn       | Knot vector (effective partition)                       |
| degree   | Spline degree (should be 1)                             |
| x_values | Evaluation points for design matrix (0 = no evaluation) |
| verbose  | logical; if TRUE, print progress messages               |

**Value**

A list containing:

|    |                                                 |
|----|-------------------------------------------------|
| d0 | Design matrix (if x_values provided)            |
| d1 | First derivative values (constant per interval) |

---

bspline\_to\_deriv\_coeffs\_quad  
*quadratic B-spline derivative coefficients*

---

**Description**

Computes normalized first and second derivative coefficients for quadratic B-splines on each interval.

**Usage**

```
bspline_to_deriv_coeffs_quad(tn, degree = 2, x_values = 0, verbose = FALSE)
```

**Arguments**

|          |                                                         |
|----------|---------------------------------------------------------|
| tn       | Knot vector (effective partition)                       |
| degree   | Spline degree (should be 2)                             |
| x_values | Evaluation points for design matrix (0 = no evaluation) |
| verbose  | logical; if TRUE, print progress messages               |

**Value**

A list containing:

|    |                                                          |
|----|----------------------------------------------------------|
| d0 | Design matrix (if x_values provided)                     |
| d1 | First derivative coefficients [a1, a0] for each interval |
| d2 | Second derivative values (constant per interval)         |

---

bspline\_to\_deriv\_coeffs\_quart

*Convert quartic B-spline to derivative coefficients*


---

**Description**

Computes normalized first, second, and third derivative coefficients for quartic B-splines on each interval.

**Usage**

```
bspline_to_deriv_coeffs_quart(knot, degree = 4, x_values = 0)
```

**Arguments**

|          |                                     |
|----------|-------------------------------------|
| knot     | Knot vector (effective partition)   |
| degree   | Spline degree (should be 4)         |
| x_values | Evaluation points for design matrix |

**Value**

A list containing:

|    |                                                                  |
|----|------------------------------------------------------------------|
| d0 | Design matrix (if x_values provided)                             |
| d1 | First derivative coefficients [a3, a2, a1, a0] for each interval |
| d2 | Second derivative coefficients [a2, a1, a0] for each interval    |
| d3 | Third derivative values at knot (linear constraints)             |

---

bs\_direct

*Direct evaluation of a B-spline basis*


---

**Description**

Computes the values of all B-spline basis functions at given points.

**Usage**

```
bs_direct(Basis, x_values, verbose = FALSE)
```

**Arguments**

|          |                                 |
|----------|---------------------------------|
| Basis    | Object returned by Bspline_base |
| x_values | Vector of evaluation points     |
| verbose  | set to TRUE increases verbosity |

**Value**

Matrix of basis function values (n\_splines x length(x\_values))

---

|                               |                                                   |
|-------------------------------|---------------------------------------------------|
| change_polynomial_base_taylor | <i>Change polynomial basis (Taylor expansion)</i> |
|-------------------------------|---------------------------------------------------|

---

**Description**

Converts a polynomial expressed in the basis (t-a)^k to its representation in the basis (t-b)^k using Taylor's formula.  $P(t) = \sum c_k (t-a)^k$   $P(t) = \sum c''_k (t-b)^k$

**Usage**

change\_polynomial\_base\_taylor(coeffs\_a, a, b)

**Arguments**

- |          |                                                         |
|----------|---------------------------------------------------------|
| coeffs_a | Coefficients in basis centered at a (decreasing powers) |
| a        | Original expansion point                                |
| b        | New expansion point                                     |

**Value**

Coefficients in basis centered at b

---

|                |                                              |
|----------------|----------------------------------------------|
| get_parameters | <i>Get parameters from a callable spline</i> |
|----------------|----------------------------------------------|

---

**Description**

Get parameters from a callable spline

**Usage**

get\_parameters(x)

**Arguments**

- |   |                          |
|---|--------------------------|
| x | A callable spline object |
|---|--------------------------|

**Value**

A list with degree, knots, coefficients, and Bspline object



---

|             |                                        |
|-------------|----------------------------------------|
| make_spline | <i>Create a callable spline object</i> |
|-------------|----------------------------------------|

---

### Description

Transforms a list container 'result' from the regression result into a callable function that is the spline function with given knots and coefficients on the corresponding B-spline basis. It can be evaluated at any point of the knot range using the spline\_eval() function, while preserving access to parameters.

### Usage

```
make_spline(Bspline, verbose = FALSE)
```

### Arguments

|         |                                                                                                                                                                                                                    |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Bspline | A list containing at least (knot, coefficients, degree) like the one returned by quantile_spline or one of the degree-specific functions. If it already contains a 'spline' element, returns the object unchanged. |
| verbose | If TRUE : output the attributes of the Bspline                                                                                                                                                                     |

### Value

The same list with an additional 'spline' element containing the callable function. The function can be called as 'Bspline(x)' and has an optional 'Bvalues' parameter to accelerate multiple evaluations at the same x\_values.

---

|         |                            |
|---------|----------------------------|
| polyadd | <i>Polynomial addition</i> |
|---------|----------------------------|

---

### Description

Adds two polynomials represented by coefficients in decreasing power order.

### Usage

```
polyadd(p1, p2, verbose = FALSE)
```

### Arguments

|         |                                        |
|---------|----------------------------------------|
| p1      | First polynomial (coefficient vector)  |
| p2      | Second polynomial (coefficient vector) |
| verbose | boolean FALSE (default) or TRUE.       |

**Value**

Coefficient vector of the sum

**Examples**

```
polyadd(c(1, 1), c(1, -1)) # returns c(2, 0)
```

---

|           |                                                                                    |
|-----------|------------------------------------------------------------------------------------|
| polyderiv | <i>Polynomial derivative Computes the derivative of order der of a polynomial.</i> |
|-----------|------------------------------------------------------------------------------------|

---

**Description**

Polynomial derivative Computes the derivative of order der of a polynomial.

**Usage**

```
polyderiv(p, der = 1)
```

**Arguments**

|     |                                        |
|-----|----------------------------------------|
| p   | Coefficient vector (decreasing powers) |
| der | Derivative order (default = 1)         |

**Value**

Coefficients of the derivative polynomial

**Examples**

```
# P(x) = x^2 -> P'(x) = 2x
polyderiv(c(1, 0, 0), 1) # returns c(2, 0)
```

---

|         |                                  |
|---------|----------------------------------|
| polymul | <i>Polynomial multiplication</i> |
|---------|----------------------------------|

---

**Description**

Multiplies two polynomials represented by coefficients in decreasing power order.

**Usage**

```
polymul(p1, p2, ord = 0, verbose = FALSE)
```

**Arguments**

|         |                                                           |
|---------|-----------------------------------------------------------|
| p1      | First polynomial (coefficient vector, decreasing powers)  |
| p2      | Second polynomial (coefficient vector, decreasing powers) |
| ord     | Unused (compatibility parameter)                          |
| verbose | boolean FALSE (default) or TRUE.                          |

**Value**

Coefficient vector of the product polynomial (decreasing powers)

**Examples**

```
# (1 + x) * (1 + x) = 1 + 2x + x^2
polymul(c(1, 1), c(1, 1)) # returns c(1, 2, 1)
```

---

|           |                            |
|-----------|----------------------------|
| poly_eval | <i>Evaluate polynomial</i> |
|-----------|----------------------------|

---

**Description**

Evaluates a polynomial at one or more points.

**Usage**

```
poly_eval(p, xvalues)
```

**Arguments**

|         |                                            |
|---------|--------------------------------------------|
| p       | Coefficient vector (decreasing powers)     |
| xvalues | vector at which to evaluate the polynomial |

**Value**

Vector of same length as xvalues : polynomial values at the points xvalues

**Examples**

```
# P(x) = 1 + x + x^2
poly_eval(c(1, 1, 1), c(0, 1, 2)) # returns c(1, 3, 7)
```

---

print.callable\_spline *Print method for callable spline*

---

**Description**

Print method for callable spline

**Usage**

```
## S3 method for class 'callable_spline'  
print(x, ...)
```

**Arguments**

|     |                          |
|-----|--------------------------|
| x   | A callable spline object |
| ... | Additional arguments     |

---

print.quantile\_spline *Print method for quantile\_spline results*

---

**Description**

Print method for quantile\_spline results

**Usage**

```
## S3 method for class 'quantile_spline'  
print(x, ...)
```

**Arguments**

|     |                                    |
|-----|------------------------------------|
| x   | Result object from quantile_spline |
| ... | Additional arguments               |

**Description**

This function provides a unified interface for quantile regression with B-splines of degrees 1 to 4, with shape constraints (monotonicity, convexity, third derivative).

**Usage**

```
quantile_spline(
  xtab,
  ytab,
  knot,
  tau,
  degree = 3,
  monot = 0,
  convcons = 0,
  der3cons = 0,
  solver = "HIGHS",
  weight = NULL,
  verbose = FALSE,
  callable = TRUE
)
```

**Arguments**

|          |                                                                                                                                                                                                                                                  |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| xtab     | Predictor vector (x)                                                                                                                                                                                                                             |
| ytab     | Response vector (y)                                                                                                                                                                                                                              |
| knot     | Knot vector or number of knots (quantiles are then used)                                                                                                                                                                                         |
| tau      | Quantile (between 0 and 1)                                                                                                                                                                                                                       |
| degree   | Spline degree: 1 (linear), 2 (quadratic), 3 (cubic), or 4 (quartic)                                                                                                                                                                              |
| monot    | Monotonicity constraint vector per interval: 1 = increasing, -1 = decreasing, 0 = unconstrained. If scalar, repeated.                                                                                                                            |
| convcons | Convexity constraint vector: For degree 1: not available (ignored) For degree 2: per interval (1 = convex, -1 = concave) For degree 3: per knot (1 = convex, -1 = concave) For degree 4: per interval (1 = convex, -1 = concave)                 |
| der3cons | Third derivative constraint: For degree 1: not available (third derivative = 0) For degree 2: not available (third derivative = 0) For degree 3: per interval (1 = positive, -1 = negative) For degree 4: per knot (1 = positive, -1 = negative) |
| solver   | CVXR solver to use (default = "CLARABEL")                                                                                                                                                                                                        |
| weight   | Observation weights (default = 1 for all)                                                                                                                                                                                                        |
| verbose  | logical; if TRUE, print progress messages                                                                                                                                                                                                        |
| callable | render the final container object callable: y=Bspline(x) or y=Bspline(x,Bvalues) for evaluation at x                                                                                                                                             |

**Value**

A list containing coefficients, degree, and knots

**References**

- Abbes, A. (2025). *Quantile regression with cubic polynomial splines under shape constraints with applications*. Zenodo. doi:10.5281/zenodo.16999784
- Karlin, S., & Studden, W. J. (1966). *Tchebycheff Systems: With Applications in Analysis and Statistics*. Interscience.

**See Also**

[SplineLinearQuant](#), [SplineQuadraticQuant](#), [SplineCubicQuant](#), [SplineQuarticQuant](#)

**Examples**

```
# Generate data
set.seed(42)
x <- seq(0, 1, length.out = 100)
y <- 2*x + sin(6*pi*x)/2 + rnorm(100, 0, 0.05)
knot <- quantile(x, probs = seq(0, 1, length.out = 10))

# Linear spline (degree 1) with increasing constraint
fit_lin <- quantile_spline(x, y, knot, tau = 0.5, degree = 1, monot = 1)

# Quadratic spline (degree 2) with convexity constraint
fit_quad <- quantile_spline(x, y, knot, tau = 0.5, degree = 2, convcons = 1)

# Cubic spline (degree 3) with both constraints
fit_cubic <- quantile_spline(x, y, knot, tau = 0.5, degree = 3,
                           monot = 1, convcons = 1)

# Quartic spline (degree 4) with all constraints
fit_quart <- quantile_spline(x, y, knot, tau = 0.5, degree = 4,
                           monot = 1, convcons = 1, der3cons = 1)
```

---

reduce\_pol

*Reduce polynomial*

---

**Description**

Removes leading zeros from a polynomial coefficient vector.

**Usage**

```
reduce_pol(p, verbose = FALSE)
```

**Arguments**

|         |                                                         |
|---------|---------------------------------------------------------|
| p       | Polynomial coefficient vector(coef in decreasing order) |
| verbose | boolean FALSE (default) or TRUE.                        |

**Value**

Reduced vector (without leading zeros)

**Examples**

```
reduce_pol(c(0,0, 1, 1))
```

---

SplineConstQuantRegBs1

*Wrapper function for linear spline quantile regression*

---

**Description**

This is a convenience wrapper for SplineLinearQuant.

**Usage**

```
SplineConstQuantRegBs1(
  xtab,
  ytab,
  knot,
  tau,
  monot = 0,
  solver = "HIGHS",
  weight = NULL,
  verbose = FALSE
)
```

**Arguments**

|         |                                                                                                 |
|---------|-------------------------------------------------------------------------------------------------|
| xtab    | Predictor vector (x)                                                                            |
| ytab    | Response vector (y)                                                                             |
| knot    | Knot vector or number of knots                                                                  |
| tau     | Quantile (between 0 and 1)                                                                      |
| monot   | Monotonicity constraint vector per interval: 1 = increasing, -1 = decreasing, 0 = unconstrained |
| solver  | CVXR solver to use (default = "OSQP")                                                           |
| weight  | Observation weights (default = 1 for all)                                                       |
| verbose | logical; if TRUE, print progress messages                                                       |

**Value**

Same as SplineLinearQuant

---

SplineConstQuantRegBs2

*Wrapper function for quadratic spline quantile regression Previous version notation style This is a convenience wrapper for SplineQuadraticQuant.*

---

**Description**

Wrapper function for quadratic spline quantile regression Previous version notation style This is a convenience wrapper for SplineQuadraticQuant.

**Usage**

```
SplineConstQuantRegBs2(
  xtab,
  ytab,
  knot,
  tau,
  monot = 0,
  convcons = 0,
  solver = "HIGHS",
  weight = NULL,
  verbose = FALSE
)
```

**Arguments**

|          |                                                                                                                                                                                                                                              |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| xtab     | Predictor vector (x)                                                                                                                                                                                                                         |
| ytab     | Response vector (y)                                                                                                                                                                                                                          |
| knot     | Knot vector or number of knots                                                                                                                                                                                                               |
| tau      | Quantile (between 0 and 1)                                                                                                                                                                                                                   |
| monot    | Monotonicity constraint vector per interval: 1 = increasing, -1 = decreasing, 0 = unconstrained. Although tconstraints are set at knots; we apply them on each interval, at both extremities. kn+1 knots, kn constraints taken into account. |
| convcons | Convexity constraint vector per interval: 1 = convex, -1 = concave, 0 = unconstrained. kn constraints are considered for kn+1 knots.                                                                                                         |
| solver   | CVXR solver to use (default = "OSQP")                                                                                                                                                                                                        |
| weight   | Observation weights (default = 1 for all)                                                                                                                                                                                                    |
| verbose  | logical; if TRUE, print progress messages                                                                                                                                                                                                    |

**Value**

Same as SplineQuadraticQuant



---

SplineConstQuantRegBs3

*Wrapper function for Cubic spline quantile regression*


---

## Description

This is a convenience wrapper for SplineCubicQuant with convention of previous versions SplineConstQuantRegBs3.

## Usage

```
SplineConstQuantRegBs3(
  xtab,
  ytab,
  knot,
  tau,
  monot = 0,
  convcons = 0,
  der3cons = 0,
  solver = "CLARABEL",
  weight = NULL,
  verbose = FALSE
)
```

## Arguments

|          |                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------|
| xtab     | Predictor vector (x)                                                                                                  |
| ytab     | Response vector (y)                                                                                                   |
| knot     | Knot vector or number of knot (quantiles are then used)                                                               |
| tau      | Quantile (between 0 and 1)                                                                                            |
| monot    | Monotonicity constraint vector per interval: 1 = increasing, -1 = decreasing, 0 = unconstrained. If scalar, repeated. |
| convcons | Convexity constraint vector per knot: 1 = convex, -1 = concave, 0 = unconstrained. If scalar, repeated.               |
| der3cons | Constraint on the 3rd derivative (on esach intervall: -1: négative, 0: no constraint, 1: positive constraint)         |
| solver   | CVXR solver to use (default = "CLARABEL")                                                                             |
| weight   | Observation weights (default = 1 for all)                                                                             |
| verbose  | boolean FALSE (default) or TRUE.                                                                                      |

## Value

Same as SplineCubicQuant

---

SplineConstQuantRegBs4

*Wrapper function for quartic spline quantile regression*

---

**Description**

This is a convenience wrapper for SplineQuarticQuant that follows the same naming convention as SplineConstQuantRegBs3.

**Usage**

```
SplineConstQuantRegBs4(
  xtab,
  ytab,
  knot,
  tau,
  monot = 0,
  convcons = 0,
  der3cons = 0,
  solver = "CLARABEL",
  weight = NULL,
  verbose = FALSE
)
```

**Arguments**

|          |                                                                                                             |
|----------|-------------------------------------------------------------------------------------------------------------|
| xtab     | Predictor vector (x)                                                                                        |
| ytab     | Response vector (y)                                                                                         |
| knot     | Knot vector or number of knot                                                                               |
| tau      | Quantile (between 0 and 1)                                                                                  |
| monot    | Monotonicity constraint vector per interval: 1 = increasing, -1 = decreasing, 0 = unconstrained             |
| convcons | Convexity constraint vector per interval: 1 = convex, -1 = concave, 0 = unconstrained                       |
| der3cons | Third derivative constraint vector at knot: 1 = positive third derivative, -1 = negative, 0 = unconstrained |
| solver   | CVXR solver to use (default = "OSQP")                                                                       |
| weight   | Observation weights (default = 1 for all)                                                                   |
| verbose  | Logical; if TRUE, print progress messages                                                                   |

**Value**

Same as SplineQuarticQuant

SplineCubicQuant

*Constrained quantile regression with cubic splines***Description**

Performs quantile regression using cubic B-splines, with optional monotonicity constraints (via Karlin-Studden) and convexity constraints.

**Usage**

```
SplineCubicQuant(
  xtab,
  ytab,
  knot,
  tau,
  monot = 0,
  convcons = 0,
  der3cons = 0,
  solver = "CLARABEL",
  weight = NULL,
  verbose = FALSE
)
```

**Arguments**

|          |                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------|
| xtab     | Predictor vector (x)                                                                                                  |
| ytab     | Response vector (y)                                                                                                   |
| knot     | Knot vector or number of knot (quantiles are then used)                                                               |
| tau      | Quantile (between 0 and 1)                                                                                            |
| monot    | Monotonicity constraint vector per interval: 1 = increasing, -1 = decreasing, 0 = unconstrained. If scalar, repeated. |
| convcons | Convexity constraint vector per knot: 1 = convex, -1 = concave, 0 = unconstrained. If scalar, repeated.               |
| der3cons | Constraint on the 3rd derivative (on each interval: -1: négative, 0: no constraint, 1: positive constraint)           |
| solver   | CVXR solver to use (default = "CLARABEL")                                                                             |
| weight   | Observation weights (default = 1 for all)                                                                             |
| verbose  | boolean FALSE (default) or TRUE.                                                                                      |

**Value**

A list containing:

coefficients    B-spline coefficients (including y mean)

|          |                          |
|----------|--------------------------|
| degree   | Spline degree (always 3) |
| ext_knot | ext_knot vector used     |
| int_knot | knot vector              |

## References

- Abbes, A. (2025). *Quantile regression with cubic polynomial splines under shape constraints with applications*. Zenodo. doi:10.5281/zenodo.16999784
- de Boor, C. (1978). *A Practical Guide to Splines*. Springer-Verlag. doi:10.1007/97814612-63333
- Karlin, S., & Studden, W. J. (1966). *Tchebycheff Systems: With Applications in Analysis and Statistics*. Interscience.
- Koenker, R., & Bassett, G. (1978). Regression Quantiles. *Econometrica*, 46(1), 33-50. doi:10.2307/1913643
- Koenker, R. (2025). quantreg: Quantile Regression. R package version 5.99. <https://CRAN.R-project.org/package=quantreg>
- Ng, P., & Maechler, M. (2024). cobs: Constrained B-Splines. R package version 1.3-8. <https://CRAN.R-project.org/package=cobs>

## See Also

Related R packages:

- quantreg - Quantile regression with linear programming
- cobs - Constrained B-sines (linear and quadratic only)

Other implementations:

- MATLAB/Python versions: <https://github.com/alexandreabbes/Constrained-Quantile-Regression-with-c>

## Examples

```
#optional set.seed(42)
x <- seq(0, 1, length=100)
y <- 2*x + sin(6*pi*x)/2 + rnorm(100, 0, 0.05)
knot <- quantile(x, probs=seq(0,1,length.out=10))

# Median quantile regression without constraints
fit <- SplineConstQuantRegBs3(x, y, knot, tau=0.5)

# With increasing monotonicity constraint
fit_monot <- SplineConstQuantRegBs3(x, y, knot, tau=0.5, monot=1)

# With convexity constraint
fit_convex <- SplineConstQuantRegBs3(x, y, knot, tau=0.5, convcons=1)
```

---

|                   |                                                                             |
|-------------------|-----------------------------------------------------------------------------|
| SplineLinearQuant | <i>Quantile regression with linear splines and monotonicity constraints</i> |
|-------------------|-----------------------------------------------------------------------------|

---

## Description

Performs quantile regression using linear B-splines with monotonicity constraints. For linear splines, the derivative is constant on each interval, so monotonicity is a simple linear constraint on the derivative.

## Usage

```
SplineLinearQuant(
  xtab,
  ytab,
  knot,
  tau,
  monot = 0,
  solver = "GUROBI",
  weight = NULL,
  verbose = FALSE
)
```

## Arguments

|         |                                                                                                 |
|---------|-------------------------------------------------------------------------------------------------|
| xtab    | Predictor vector (x)                                                                            |
| ytab    | Response vector (y)                                                                             |
| knot    | Knot vector or number of knots                                                                  |
| tau     | Quantile (between 0 and 1)                                                                      |
| monot   | Monotonicity constraint vector per interval: 1 = increasing, -1 = decreasing, 0 = unconstrained |
| solver  | CVXR solver to use (default = "OSQP")                                                           |
| weight  | Observation weights (default = 1 for all)                                                       |
| verbose | logical; if TRUE, print progress messages                                                       |

## Value

A list containing coefficients, degree, and knots

---

SplineQuadraticQuant    *Quantile regression with quadratic splines and shape constraints*


---

### Description

Performs quantile regression using quadratic B-splines with: - Monotonicity: Karlin-Studden constraints on the derivative (linear) - Convexity: Karlin-Studden constraints on the second derivative (constant)

### Usage

```
SplineQuadraticQuant(
  xtab,
  ytab,
  knot,
  tau,
  monot = 0,
  convcons = 0,
  solver = "GUROBI",
  weight = NULL,
  verbose = FALSE
)
```

### Arguments

|          |                                                                                                                                                                                                                                              |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| xtab     | Predictor vector (x)                                                                                                                                                                                                                         |
| ytab     | Response vector (y)                                                                                                                                                                                                                          |
| knot     | Knot vector or number of knots                                                                                                                                                                                                               |
| tau      | Quantile (between 0 and 1)                                                                                                                                                                                                                   |
| monot    | Monotonicity constraint vector per interval: 1 = increasing, -1 = decreasing, 0 = unconstrained. Although tconstraints are set at knots; we apply them on each interval, at both extremities. kn+1 knots, kn constraints taken into account. |
| convcons | Convexity constraint vector per interval: 1 = convex, -1 = concave, 0 = unconstrained. kn constraints are considered for kn+1 knots.                                                                                                         |
| solver   | CVXR solver to use (default = "OSQP")                                                                                                                                                                                                        |
| weight   | Observation weights (default = 1 for all)                                                                                                                                                                                                    |
| verbose  | logical; if TRUE, print progress messages                                                                                                                                                                                                    |

### Value

A list containing coefficients, degree, and knots

---

|                    |                                                                       |
|--------------------|-----------------------------------------------------------------------|
| SplineQuarticQuant | <i>Quantile regression with quartic splines and shape constraints</i> |
|--------------------|-----------------------------------------------------------------------|

---

**Description**

Performs quantile regression using quartic (degree 4) B-splines with monotonicity (Karlin-Studden constraints on the cubic derivative), convexity (Karlin-Studden constraints on the quadratic second derivative), and third derivative constraints (linear at knot).

**Usage**

```
SplineQuarticQuant(
  xtab,
  ytab,
  knot,
  tau,
  monot = 0,
  convcons = 0,
  der3cons = 0,
  solver = "CLARABEL",
  weight = NULL,
  verbose = FALSE
)
```

**Arguments**

|          |                                                                                                             |
|----------|-------------------------------------------------------------------------------------------------------------|
| xtab     | Predictor vector (x)                                                                                        |
| ytab     | Response vector (y)                                                                                         |
| knot     | Knot vector or number of knot                                                                               |
| tau      | Quantile (between 0 and 1)                                                                                  |
| monot    | Monotonicity constraint vector per interval: 1 = increasing, -1 = decreasing, 0 = unconstrained             |
| convcons | Convexity constraint vector per interval: 1 = convex, -1 = concave, 0 = unconstrained                       |
| der3cons | Third derivative constraint vector at knot: 1 = positive third derivative, -1 = negative, 0 = unconstrained |
| solver   | CVXR solver to use (default = "OSQP")                                                                       |
| weight   | Observation weights (default = 1 for all)                                                                   |
| verbose  | Logical; if TRUE, print progress messages                                                                   |

**Value**

A list containing coefficients, degree, and knot

---

|                 |                                          |
|-----------------|------------------------------------------|
| Spline_der_knot | <i>Derivatives at knot of a B-spline</i> |
|-----------------|------------------------------------------|

---

**Description**

Computes derivative values of a B-spline at knot (efficient because it directly uses polynomial coefficients).

**Usage**

```
Spline_der_knot(Bsbase, der = 1)
```

**Arguments**

|        |                                 |
|--------|---------------------------------|
| Bsbase | Object returned by Bspline_base |
| der    | Derivative order (default = 1)  |

**Value**

Matrix of derivative values (n\_splines x n\_knot)

---

|             |                            |
|-------------|----------------------------|
| spline_eval | <i>Evaluate a B-spline</i> |
|-------------|----------------------------|

---

**Description**

Evaluates a spline (linear combination of B-splines) at given points.

**Usage**

```
spline_eval(Bspline, x_values = NULL, Bvalues = NULL)
```

**Arguments**

|          |                                                                                                                                                                                   |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Bspline  | Spline object (list with coefficients on the Bspline basis, degree, extended knot)<br>A Bspline can also be rendered callable with make_spline(BSpline)                           |
| x_values | Vector of evaluation points. By default, evaluation is calculated at the knots                                                                                                    |
| Bvalues  | : the values of the Bspline basis evaluated at the values x this increases the speed by avoiding re-doing the same calculations of the basis for each spline with the same knots. |

**Value**

vector of same length as x\_values, with Spline values at the requested points



**Examples**

```
{ # Create and evaluate a spline
# This function is also used when a Bspline object is rendered callable as a function
# with meake_spline()
tn<-c(0,1,2,3,4,5)
x_values<-seq(0,5,length=100)
Bspline=list(degree=3, knot=tn,coefficients=runif(length(tn)+3-1))
y <- spline_eval(Bspline, x_values)
#alternatively compute the base before to optimize if needed
sn <- c(0,0,0,0,1,2,3,4,5,5,5,5)
Bsbase <- Bspline_base(sn, degree=3)
Bvalues <-bs_direct(Bsbase,x_values)
y <- spline_eval(Bspline=Bspline,x_values=x_values, Bvalues=Bvalues )}
```

view\_basis

*Visualize a B-spline functions basis***Description**

Plots all functions of a B-spline basis.

**Usage**

```
view_basis(BB, x_values = 0)
```

**Arguments**

|          |                                                                                                 |
|----------|-------------------------------------------------------------------------------------------------|
| BB       | Object returned by Bspline_base                                                                 |
| x_values | Vector of evaluation points for plotting (by default 100 points are computed in the knot range) |

**Value**

No return value, called for side effects (generates a plot)

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